Eur. Phys. J. C 26, 45–49 (2002)
Digital Object Identifier (DOI) 10.1140/epjc/s2002-01041-2

THE EUROPEAN PHYSICAL JOURNAL C

Spin structure function $g_1(x, Q^2)$ and the DHGHY integral $I(Q^2)$ at low Q^2: predictions from the GVMD model

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Received: 9 July 2002 /
Published online: 7 October 2002 –

Abstract. Theoretical predictions for the polarized nucleon structure function $g_1(x, Q^2)$ at low Q^2 are obtained in the framework of the generalized vector meson dominance model. Contributions from both light and heavy vector mesons are evaluated. In the photoproduction limit the first moment of g_1 is related to the static properties of the nucleon via the Drell–Hearn–Gerasimov–Hosoda–Yamamoto sum rule. This property is employed to fix the magnitude of the light vector meson contribution to g_1, using the recent measurements in the region of baryonic resonances. The results are compared to the data on $g_1(x, Q^2)$. Finally, the DHGHY moment function $I(Q^2)$ is calculated, and our theoretical predictions are confronted with the recent preliminary data obtained at the Jefferson Laboratory.

1 Introduction

Data on the polarized nucleon structure function $g_1(x, Q^2)$ are now available in the region of low values of (negative) four-momentum transfer, Q^2 [1–4]. This region is of particular interest since non-perturbative mechanisms dominate the particle dynamics there and thus a transition from soft to hard physics may be studied. In the fixed target experiments the low values of Q^2 are reached simultaneously with the low values of the Bjorken variable, x [2, 3], and therefore predictions for the spin structure functions in both the low x and low Q^2 region are needed. The partonic contribution to g_1, which controls the structure function in the deep inelastic domain, thus has to be suitably extended to the low Q^2 region. In the $Q^2 = 0$ limit, g_1 should be a finite function of W^2, free from any kinematic singularities or zeroes.

In a previous attempt [5], g_1 at low x and low Q^2 was described within a formalism based on the unintegrated spin dependent parton distributions, incorporating the leading order Altarelli–Parisi evolution and the double $\ln^2(1/x)$ resummation at low x. The $\ln^2(1/x)$ effects are not yet significant in the kinematic range of the fixed target experiments, but the formalism based on unintegrated parton distributions is very suitable for extrapolating g_1 to the region of low Q^2. Also a VMD-type non-perturbative part of g_1 was included, its unknown normalization to be extracted from the data. The model reproduced general trends in the measurements, but their statistical accuracy was too low to constrain the VMD contribution. However the non-zero and negative value of the VMD contribution was clearly preferred.

A convenient way of describing structure functions both in the non-perturbative and in the scaling ("asymptotic") region is to employ the generalized vector meson dominance (GVMD) model with an infinite number of vector mesons which couple to a virtual photon (cf. [6]). The heavy meson ($M_V > Q_0$) contribution is directly related to the structure function in the scaling region, g_1^{AS}, described by the QCD improved parton model, suitably extrapolated to the low Q^2 region. The contribution of light ($M_V < Q_0$) vector mesons describes non-perturbative effects and vanishes as $1/Q^4$ for large Q^2. At low Q^2 these effects are large and predominant. Here M_V denotes the mass of the vector meson. The GVMD model was successfully applied to describe the low Q^2 behavior of the unpolarized structure function $F_2(x, Q^2)$ [7,8].

In this paper we apply GVMD to evaluate the non-perturbative contributions to the polarized structure function $g_1(x, Q^2)$ at low values of Q^2. We start with the for-

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mulation of the GVMD representation of $g_1(x,Q^2)$ and then specify the representation of the light and of the heavy vector meson components: vector meson dominance (VMD) for the former and the asymptotic approach for the latter one (Sect. 2). Two different parametrizations are used to describe the VMD part of $g_1(x,Q^2)$. The asymptotic part is parametrized using the GRSV fit [9]. Then the Drell–Hearn–Gerasimov–Hosoda–Yamamoto (DHGHY) sum rule [10–12] together with measurements in the resonance region are employed to fix the magnitude of the light vector meson contribution to g_1 (Sect. 2). Numerical results are discussed and compared to other analyses in Sect. 3. Conclusions and an outlook are given in Sect. 4.

2 The GVMD representation of the structure function $g_1(x,Q^2)$ and the DHGHY sum rule

In the GVMD model, the spin dependent nucleon structure function g_1 has the following representation, valid for fixed $W^2 \gg Q^2$, i.e. small values of x, $x = Q^2/(Q^2+W^2-M^2)$:

$$g_1(x,Q^2) = g_1^{\mathrm{L}}(x,Q^2) + g_1^{\mathrm{H}}(x,Q^2). \tag{1}$$

The first term,

$$g_1^{\mathrm{L}}(x,Q^2) = \frac{M\nu}{4\pi}\sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2(Q^2+M_V^2)^2}, \tag{2}$$

sums up contributions from light vector mesons, $M_V < Q_0$, where $Q_0^2 \sim 1\,\mathrm{GeV}^2$ [8]. Here W is the invariant mass of the electroproduced hadronic system, $\nu = Q^2/2Mx$, and M denotes the nucleon mass. The constants γ_V^2 are determined from the leptonic widths of the vector mesons and the cross sections $\Delta\sigma_V(W^2)$ are combinations of the total cross sections for the scattering of polarized mesons and nucleons. They are not known and have to be parametrized. Following [5], we assume that they can be expressed through the combinations of non-perturbative parton distributions, $\Delta p_j^{(0)}(x)$, evaluated at fixed Q_0^2:

$$\begin{aligned}
&\frac{M\nu}{4\pi}\sum_{V=\rho,\omega} \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2(Q^2+M_V^2)^2} \\
&= C\left[\frac{4}{9}(\Delta u_{\mathrm{val}}^{(0)}(x) + \Delta\bar{u}^{(0)}(x))\right. \\
&+ \left.\frac{1}{9}(\Delta d_{\mathrm{val}}^{(0)}(x) + \Delta\bar{d}^{(0)}(x))\right]\frac{M_\rho^4}{(Q^2+M_\rho^2)^2}, \\
&\frac{M\nu}{4\pi}\frac{M_\phi^4\Delta\sigma_\phi(W^2)}{\gamma_\phi^2(Q^2+M_\phi^2)^2} \\
&= C\left[\frac{1}{9}(2\Delta\bar{s}^{(0)}(x))\right]\frac{M_\phi^4}{(Q^2+M_\phi^2)^2}.
\end{aligned} \tag{3}$$

It should be noted that the magnitude of the VMD contribution to $g_1(x,Q^2)$ in (1) is weighted by an unknown constant C.

The second term in (1), $g_1^{\mathrm{H}}(x,Q^2)$, which represents the contribution of heavy ($M_V > Q_0$) vector mesons to $g_1(x,Q^2)$ can also be treated as an extrapolation of the QCD improved parton model structure function, $g_1^{\mathrm{AS}}(x, Q^2)$, to arbitrary values of Q^2. We shall use a simplified representation of $g_1^{\mathrm{H}}(x,Q^2)$,

$$g_1^{\mathrm{H}}(x,Q^2) = g_1^{\mathrm{AS}}(\bar{x},Q^2+Q_0^2), \tag{4}$$

as was done in [8] for the unpolarized structure function F_2. The scaling variable x on the right hand side of (4) is replaced by $\bar{x} = (Q^2+Q_0^2)/(Q^2+Q_0^2+W^2-M^2)$. It follows from (4) that $g_1^{\mathrm{H}}(x,Q^2) \to g_1^{\mathrm{AS}}(x,Q^2)$ as Q^2 is large. Substituting $g_1^{\mathrm{H}}(x,Q^2)$ in (1) with (4) and $g_1^{\mathrm{L}}(x,Q^2)$ with the sum of (3) we get

$$\begin{aligned}
g_1(x,Q^2) &= g_1^{\mathrm{L}}(x,Q^2) + g_1^{\mathrm{AS}}(\bar{x},Q^2+Q_0^2) \\
&= C\left[\frac{4}{9}(\Delta u_{\mathrm{val}}^{(0)}(x) + \Delta\bar{u}^{(0)}(x))\right. \\
&+ \left.\frac{1}{9}(\Delta d_{\mathrm{val}}^{(0)}(x) + \Delta\bar{d}^{(0)}(x))\right]\frac{M_\rho^4}{(Q^2+M_\rho^2)^2} \\
&+ C\left[\frac{1}{9}(2\Delta\bar{s}^{(0)}(x))\right]\frac{M_\phi^4}{(Q^2+M_\phi^2)^2} \\
&+ g_1^{\mathrm{AS}}(\bar{x},Q^2+Q_0^2).
\end{aligned} \tag{5}$$

The only free parameter in (5) is the constant C. Its value may be fixed in the photoproduction limit where the first moment of the structure function $g_1(x,Q^2)$ is related to static properties of the nucleon via the DHGHY sum rule, cf. [13,14]. These static properties are the anomalous magnetic moments. For a photon frequency ν defined by $\nu = pq/M$ (p and q are energy-momentum four-vectors of the target nucleon and virtual photon respectively, and $q^2 = -Q^2$), the photon–nucleon scattering amplitude, $S_1(\nu,q^2)$, fulfills the dispersion relation [13]:

$$S_1(\nu,q^2) = 4\int_{-q^2/2M}^{\infty} \nu'\mathrm{d}\nu' \frac{G_1(\nu',q^2)}{(\nu')^2-\nu^2}, \tag{6}$$

where $G_1(\nu,q^2)$ is the polarized nucleon structure function, which in the Bjorken limit (Q^2, $\nu\to\infty$ at fixed x) is related to $g_1(x,Q^2)$ by

$$G_1(\nu,q^2) = \frac{M}{\nu}g_1(x,Q^2). \tag{7}$$

As a result of Low's theorem [15],

$$S_1(0,0) = -\kappa_{p(n)}^2, \tag{8}$$

the function G_1 fulfills the DHGHY sum rule in the photoproduction limit, $Q^2 \to 0$:

$$\int_0^\infty \frac{\mathrm{d}\nu}{\nu}G_1(\nu,0) = -\frac{1}{4}\kappa_{p(n)}^2. \tag{9}$$

Here $\kappa_{p(n)}$ is the anomalous magnetic moment of the proton (or neutron). In the limit $\nu \to 0$ the right hand side of (6) transforms to

$$S_1(0,q^2) = 4M\int_{Q^2/2M}^{\infty} \frac{\mathrm{d}\nu}{\nu^2} g_1\left(x(\nu),Q^2\right). \tag{10}$$

The representation (10) of the scattering amplitude $S_1(0,q^2)$ is valid down to certain threshold value of W_t, $W_t \gtrsim 2\,\mathrm{GeV}$. Below W_t the scattering is dominated by baryonic resonances. We have to separate these two regions in (10), cf. [13,14]. The requirement $W > W_t$ gives the lower limit for integration over ν in (10): $\nu > \nu_t(Q^2)$, where $\nu_t(Q^2) = (W_t^2 + Q^2 - M^2)/2M$. With the DHGHY moment defined by $I(Q^2) = S_1(0,q^2)/4$, relation (10) may be rewritten

$$I(Q^2) = I_{\mathrm{res}}(Q^2) + M\int_{\nu_t(Q^2)}^{\infty} \frac{\mathrm{d}\nu}{\nu^2} g_1\left(x(\nu), Q^2\right), \quad (11)$$

and the DHGHY sum rule implies that

$$I(0) = I_{\mathrm{res}}(0) + M\int_{\nu_t(0)}^{\infty} \frac{\mathrm{d}\nu}{\nu^2} g_1\left(x(\nu), 0\right) = -\kappa^2_{p(n)}/4. \quad (12)$$

Here $I_{\mathrm{res}}(Q^2)$ denotes the contribution from resonances. Substituting $g_1\left(x(\nu),0\right)$ in (12) by (5) at $Q^2 = 0$ and performing the integral in (12), we may obtain the value of the constant C if $I_{\mathrm{res}}(0)$ is known e.g. from measurements.

3 Numerical calculations for the proton

To obtain the value of C from (12), the contribution of resonances was evaluated using the preliminary data taken at ELSA/MAMI by the GDH Collaboration [16] at the photoproduction, for $W_t = 1.8\,\mathrm{GeV}$. The asymptotic part of g_1 was parametrized using the GRSV2000 fit for the "standard scenario" of polarized parton distributions with a flavor symmetric light sea, $\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s}$, at the NLO accuracy [9]. The non-perturbative parton distributions, $\Delta p_j^{(0)}(x)$, in the light vector meson component of g_1, (3), were evaluated at fixed $Q^2 = Q_0^2$, using, either
(i) the GRSV2000 fit, or
(ii) a simple, "flat" input:

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i}, \quad (13)$$

with η_{u_v}=η_{d_v}=3, $\eta_{\bar{u}}$=$\eta_{\bar{s}}$=7 and η_g=5. The normalization constants N_i were determined by imposing the Bjorken sum rule for $\Delta u_v^{(0)} - \Delta d_v^{(0)}$, and requiring that the first moments of all other distributions are the same as those determined from the QCD analysis [18]. It was checked that the parametrization (13) combined with the unified equations gives a reasonable description of the SMC data on $g_1^{\mathrm{NS}}(x,Q^2)$ [19] and on $g_1^p(x,Q^2)$ [5]. This fit was also used to investigate the magnitude of the double logarithmic corrections, $\ln^2(1/x)$, to the spin structure function of the proton at low x [20]. We have assumed $Q_0^2 = 1.2\,\mathrm{GeV}^2$, cf. (1) and (3), in accordance with the analysis of F_2 [7,8]. As a result the constant C was found to be -0.30 in case (i) and -0.24 in case (ii). These values change at most by 13% when Q_0^2 changes in the interval $1.0 < Q_0^2 < 1.6\,\mathrm{GeV}^2$.

A negative value of the non-perturbative, vector meson dominance, contribution was also obtained within a formalism based on unintegrated spin dependent parton

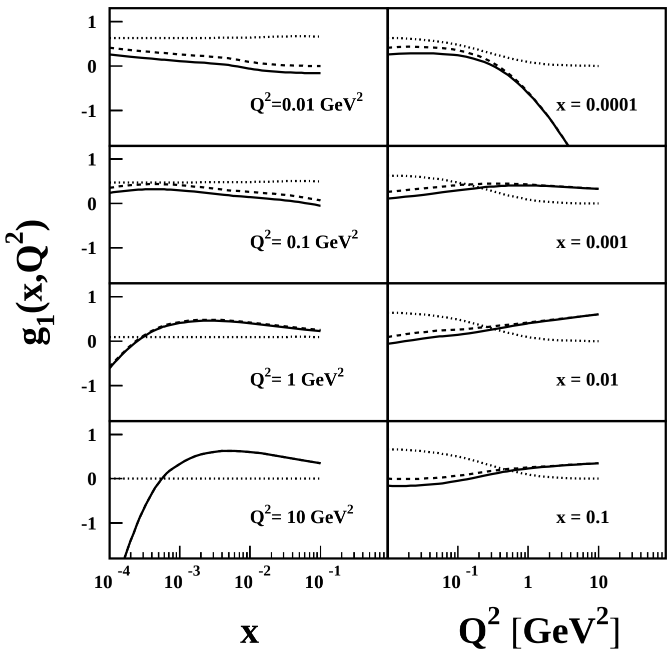

Fig. 1. Values of g_1 for the proton as a function of x and Q^2. The asymptotic contribution, g_1^{AS}, is marked with broken lines, the VMD part, g_1^{L}, with dotted lines and the continuous curves mark their sum, according to (5)

distributions supplemented with the VMD [5] and from the phenomenological analysis of the sum rules [14,22].

Using the above values of C, we have calculated $g_1(x,Q^2)$ from (5). The results are shown in Fig. 1 for different values of x and Q^2, and separately for the VMD and asymptotic parts. The VMD decreases fast with Q^2; for $Q^2 \sim 10\,\mathrm{GeV}^2$ the asymptotic function coincides to better than 0.1% with $g_1(x,Q^2)$.

The only measurements performed at low values of x and Q^2 are those obtained by the SMC with a dedicated low x trigger, [3]. They are plotted in Fig. 2, together with the results of our model, calculated at the (x,Q^2) values corresponding to those of the SMC. The mean Q^2 at lowest (highest) x is $0.02\,\mathrm{GeV}^2$ ($0.63\,\mathrm{GeV}^2$). The kinematic region of that experiment corresponds to $x < x_t = Q^2/2M\nu_t(Q^2)$, i.e. it lies outside the baryonic resonance region. Our model well reproduces a general trend in the data; however the experimental errors are too large for a more detailed analysis.

We have also computed the DHGHY moment, (11), for the proton. As a resonance input we used the preliminary results of the JLAB E91-023 experiment [17] for $0.15 \lesssim Q^2 \lesssim 1.2\,\mathrm{GeV}^2$ and $W < W_t = W_t(Q^2)$ [21]. The results are shown in Fig. 3, separately for contributions from the VMD, from the asymptotic part and from the resonances. Partons contribute significantly even in the photoproduction limit where the main part of the $I(Q^2)$ comes from resonances.

In Fig. 4 we show our DHGHY moment together with the results of calculations of [22,23] as well as with the SLAC and E91-023 measurements in the resonance region; the latter were used as an input to our $I(Q^2)$ calculations.

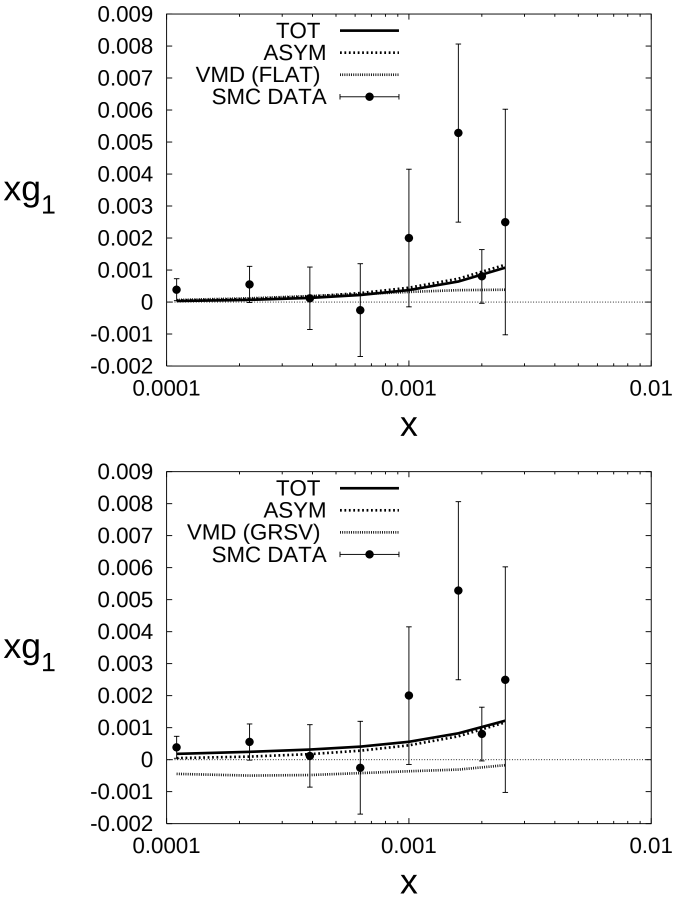

Fig. 2. Values of xg_1 for the proton as a function of x at the measured values of Q^2 in the non-resonant region, $x < x_t = Q^2/2M\nu_t(Q^2)$. The upper plot corresponds to the VMD part parametrized using (13), the lower plot corresponds to the GRSV parametrization [9] of the VMD input. The g_1^{AS} in both plots has been calculated using the GRSV fit for standard scenario at the NLO accuracy. The contributions of the VMD and of the xg_1^{AS} are shown separately. Points are the SMC measurements at $Q^2 < 1\,\mathrm{GeV}^2$ [3]; errors are total. The curves have been calculated at the measured x and Q^2 values

We also show the E91-023 data corrected by their authors for the deep inelastic contribution. Our calculations are slightly larger than the DIS-corrected data and the results of [22] but clearly lower than the results of [23] which overshoot the data.

4 Conclusions and outlook

We have analyzed the spin dependent structure function $g_1(x, Q^2)$ at low values of Q^2 and x in the framework of the generalised vector meson dominance model. This model was very successful in describing the behavior of the unpolarized structure function F_2 in the same kinematic region. Contributions from both light and heavy vector mesons have been evaluated. The latter part of the structure function is directly related to the (QCD improved) parton model g_1 and was evaluated using the GRSV2000

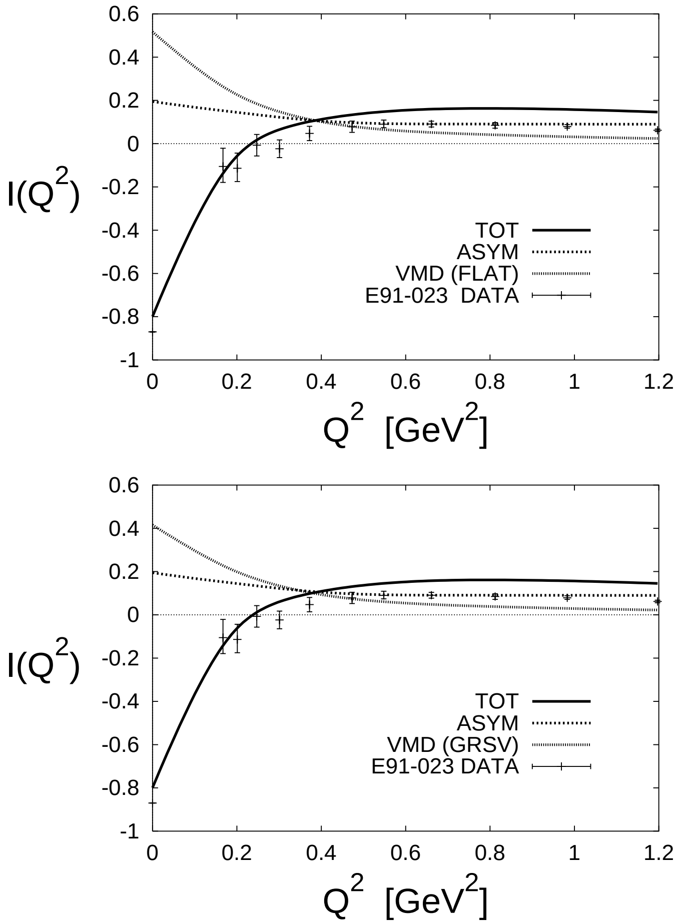

Fig. 3. The DHGHY moment $I(Q^2)$ for the proton. Details concerning the curves are as in Fig. 2. Points give the contribution of resonances as measured by the JLAB E91-023 experiment [17] at $W < W_t(Q^2)$

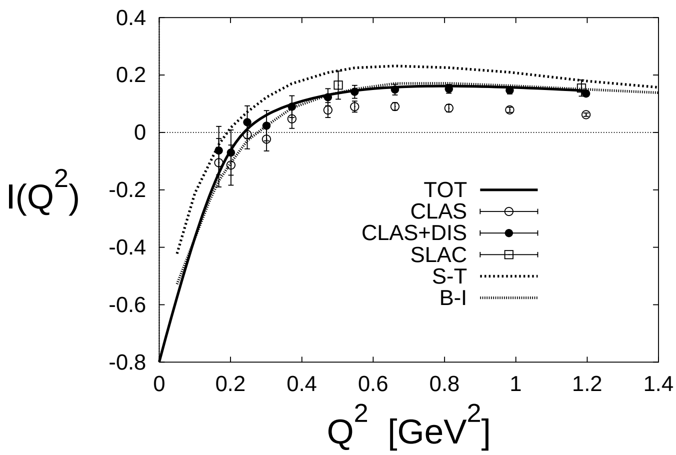

Fig. 4. The DHGHY moment $I(Q^2)$ for the proton with the VMD part parametrized using the GRSV fit [9]. Shown are also calculations of [22] ("B–I") and [23] ("S–T"). Points marked "CLAS" result from the JLAB E91-023 experiment using the Cebaf Large Angle Spectrometer, CLAS [17]: the open circles refer to the resonance region, $W < W_t(Q^2)$, and the full circles contain a correction for the DIS contribution. Points marked "SLAC" are from [1]. Errors are total

fit in the standard scenario at the NLO accuracy. The light meson contribution represents the non-perturbative part of g_1 and was parametrized either through combinations of the GRSV2000 parton distributions evaluated at fixed $Q^2 = Q_0^2$, or through combinations of (almost) x-independent phenomenological parton distributions. The contribution from the non-perturbative part was fixed using the Drell–Hearn–Gerasimov–Hosoda–Yamamoto sum rule which is related to the first moment of g_1 in the photoproduction limit. To employ that property independent information about the resonance region was necessary. We used preliminary measurements of the GDH Collaboration at ELSA/MAMI and of the JLAB E91-023 experiment. As a result the contribution of the non-perturbative part of g_1 was found to be negative and equal to about 0.24–0.30, depending on the parametrization. This result changes by at most 13% for the Q^2 in the interval $1.0 < Q^2 < 1.6\,\mathrm{GeV}^2$. The final spin dependent structure function $g_1(x, Q^2)$ reproduces well the trends in the low x, low Q^2 data of the SMC, but for a more detailed comparison more precise measurements are needed. Hopefully they will soon be performed by the COMPASS experiment at CERN. We have also computed the DHGHY integral which reproduces well the DIS-corrected preliminary measurements by JLAB E91-023.

Acknowledgements. This research has been supported in part by the Polish Committee for Scientific Research with grants 2 P03B 05119, 2PO3B 14420 and European Community grant "Training and Mobility of Researchers", Network "Quantum Chromodynamics and the Deep Structure of Elementary Particles" FMRX-CT98-0194. B.Z. was supported by the Wenner-Gren Foundations.

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